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$\therefore$ Area $DOEB=(a^2b^2m-a^2bm^2+2ab^2mn-abm^2n+b^2n^2m$
$-ab^3n-b^3n^2)\sin C/[2m(ab-am+bn)]$.

The area $DOEB$ does not vary.

The average area of $GOE+DOF=\frac{1}{2}(COE+AOD)$

$$=\frac{(2abm^2n-2b^2mn^2-2ab^2mn+ab^3n+b^3n^2)\sin C}{4m(ab-am+bn)}.$$

$$\therefore\ \triangle_1=DOEB+\tfrac{1}{2}COE+AOD.$$

$$\therefore\ \triangle_1=\frac{(2a^2b^2m-2a^2bm^2+2ab^2mn-ab^3n-b^3n^2)\sin C}{4m(ab-am+bn)}.$$

Similarly, $\triangle_2=\dfrac{(2b^2c^2m-2b^2cm^2+2bc^2mn-bc^3n-c^3n^2)\sin C}{4m(cb-bm+cn)}$.

$$\triangle_3=\frac{(2a^2c^2m-2ac^2m^2+2a^2cmn-a^3cn-a^3n^2)\sin C}{4m(ac-cm+an)}.$$

$\triangle=\frac{1}{3}(\triangle_1+\triangle_2+\triangle_3)$. If $a=b=c$ and $m=2n=\frac{2}{3}a$, $\triangle=\frac{1}{8}(a^2\sqrt{3})=\frac{1}{2}$ area of the triangle.

DIOPHANTINE ANALYSIS.

65. Proposed by MANSFIELD MERRIMAN, C. E.,Ph. D., Professor of Civil Engineering, Lehigh University, South Bethlehem, Pa.

Show that the number 1521 can be expressed in seven different ways as the sum of three perfect squares. Can more than seven different ways be found?

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

With the aid of a table that I devised for finding the sum of two squares equal to a number, I found 1521 expressed in *eight* different ways as the sum of three perfect squares. $1521=39^2=2^2+19^2+34^2=2^2+26^2+29^2=9^2+12^2+36^2$ $=10^2+14^2+35^2=13^2+14^2+34^2=13^2+26^2+26^2=14^2+22^2+29^2=19^2+22^2$ $+26^2$.

II. Solution by CHARLES CARROLL CROSS, Libertytown, Md.

The terminal figures of square numbers are, 0, 1, 4, 5, 6 and 9. These may be combined,—taking three at a time, so that the terminal figure will be one in the following manner: (5+5+1)..... (1); (9+1+1)......(2); (1+0+0)..... (3); (9+6+6)......(4); (6+4+1)..... (5); and (6+5+0)...... (6). It readily appears that it is impossible to combine any square numbers represented by (1), (2) and (3) so as to make 1521.

Those of (4) are found to be $13^2+34^2+14^2$ and $13^2+26^2+26^2$; of (5), $34^2+2^2+19^2$, $26^2+22^2+19^2$, $26^2+2^2+29^2$ and $14^2+22^2+29^2$; and of (6), $14^2+36^2+10^2$,—seven in all.

$\therefore 1521=13^2+34^2+14^2=13^2+26^2+26^2=34^2+2^2+19^2=26^2+22^2+19^2$ $=26^2+2^2+29^2=14^2+22^2+29^2=14^2+35^2+10^2$.

III. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Me.

Let $1521=m^2$, and x^2, y^2, and z^2 represent the three squares ; then $x^2+y^2+z^2=m^2$, and $x^2=m^2-(y^2+z^2)=m^2-2pm+p^2$. $2pm=p^2+y^2+z^2$. Let $y=tp$ and $z=sp$, then $2m=p(s^2+t^2+1)$. Restoring the value of m, $p^2=78/(s^2+t^2+1)$, in which s and t may be any rational numbers. Take $s=1$, $t=1$, then $p=26$; $x=m-p=13$, $y=tp=26$ and $z=sp=26$, and $13^2+26^2+26^2=1521$. Take $s=2$, and $t=1$, $p=13$; $x=26$, $y=13$, $z=26$. Take $s=3$, $t=4$, then $p=3$; $x=36$, $y=12$, $z=9$. Take $s=2$, $t=3$, $p=\frac{39}{7}$; $x=\frac{234}{7}$, $y=\frac{78}{7}$, $z=\frac{117}{7}$, and $[(234)^2+(78)^2+(117)^2]/49=1521$.

While this is a solution of the question read *literally*, of course, I understand that the proposer intends to call for integral numbers ; but I have obtained seven integral results, only *by trial*.

Also solved by *SYLVESTER ROBINS*, and *G. B. M. ZERR*.

66. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Find two cubic proper fractions whose product is a square proper fraction. · Can a *general* solution be made ?

Solution by E. L. SHERWOOD, A. M., Superintendent City Schools, West Point, Miss.; CHARLES CARROLL CROSS, Libertytown, Md.; and G. B. M. ZERR, A. M., Ph. D., Chester High School, Chester, Pa.

Let a^6/b^6, c^6/d^6 be the fractions, $a<b$, $c<d$.

Then $a^6c^6/b^6d^6=(a^3c^3/b^3d^3)^2$.

Let $a=1$, $b=2$, $c=2$, $d=3$.

$\therefore a^6/b^6=\frac{1}{64}$, $c^6/d^6=\frac{64}{729}$;

$\therefore a^6/b^6=(\frac{1}{4})^3$, $c^6/d^6=(\frac{4}{9})^3$; $a^6c^6/b^6d^6=(\frac{1}{27})^2$.

Other fractions can easily be found.

Also solved by *J. H. DRUMMOND*.

67. Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Find (1) four consecutive numbers whose sum is a square, and (2) four consecutive numbers the sum of whose squares is a square.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let $x-1$, x, $x+1$ and $x+2$=four consecutive *integers*.

(1) Then their sum$=4x+2=2(2x+1)=2$ *times an odd number*. But this result can never be a square. ∴ The sum of four consecutive integers can *not* be a square.

(2) The sum of their squares$=4x^2+4x+6=2[2(x^2+x+1)+1]=2$ *times an odd number*, which result can never be a square. ∴ The sum of the squares of four consecutive integers can *not* be a square.

If, however, four consecutive numbers may be considered as four *fractions* whose denominators are the same number and equal to 2 *times a square*, and